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Vector sum method: A new method for anti-sliding stability analysis

X.R. Ge
Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan, China
State Key Laboratory of Geomechanics and Geotechnical Engineering, Wuhan, China
Institute of Geotechnical Engineering, Shanghai Jiao Tong University, Shanghai, China

ABSTRACT: A new approach, named the Vector Sum Method (VSM), is proposed to analyze the anti-sliding stability problems for slope, dam foundation, etc. It is well known that the strength reduction method is generally applied in this field. But the stress state due to the strength reduction is a virtual state rather than a real one. So the safety factor calculated based on a virtual stress state is not physical sound. Some disadvantages of slope and dam foundation stability analysis methods based on strength reduction principle are discussed. To overcome the limitation of the strength reduction method, the VSM uses the real strength parameters rather than the reduced parameters to compute the stress field. The safety factor is computed based on the real stress state and the vector sum algorithm. Several typical examinations calculated by the vector sum method are presented. The comparison between results shows that the VSM is reasonable. The safety factor is calculated by an explicit formula in 2D case while by iterative computation in 3D case after the stress fields are acquired by finite element method or other methods. The VSM can be well applied to 3D problems for its simplicity and efficiency.

Subject: Analysis techniques and design methods

Keywords: stability analysis; rock slopes and foundations; numerical modeling

1 INTRODUCTION

The anti-sliding analysis is significantly important for the stability analysis of the dam foundation, slope and underground cave. Besides its theoretical and practical meanings, the anti-sliding analysis is broadly applied to the engineering. By far the limit equilibrium method (LEM) has been the most extensively used method in this field. Since the classic Swedish method was developed by Fellenius in 1927 (Fellenius 1939), the LEM has made great progress. However, the stress distribution on the inter-slice surface and the potential sliding surface is statically indeterminate and cannot be explicitly determined without certain assumptions. The focus of LEM research for decades has been on the division of slices, the magnitude, direction and position of force on the slice interface and the expression form of safety factor, for example, the simplified Bishop method, Janbu method, Low-Karafiath method, Mongenstern-Price method, Spencer method and Sarma method. Recently, Zheng (Zheng 2007) extended the LEM with no division of slices from 2D to 3D case. Besides the LEM, the finite element method (FEM) based on strength reduction principle is also a popular method used to analyze the stability problem of slope and dam (Ducan 1996; Feng et al. 1990; Zheng et al. 2002). In the LEM and FEM-based methods, the stability is usually evaluated by the so-called safety factor calculated based on the strength reduction principle. It seems that the philosophy of safety factor and strength reduction has been extensively accepted in engineering. However, two basic problems are worthy of reconsidering. One is the rationality of the strength reduction principle and another is the superposition principle of force in the solution of anti-sliding safety factor. The strength reduction principle involves more factitious treatment. The underlying physical mechanism is not clear. Since the force is a vector, the superposition principle of vector should be hold in the solution of anti-sliding safety factor. Hence, to explore more physically sound method for sliding stability analysis, we proposed the vector sum method (VSM) (Ge 1987; Ge et al. 1995). Some advances on this method have been made recently (Liu 2007; Ge 2008).

2 DISCUSSION ON THE RATIONALITY OF STRENGTH REDUCTION PRINCIPLE

In the safety factor algorithm, a strength reduction coefficient is firstly assumed, and then the shear strength (cohesion c and tangent of friction angle $\tan\varphi$ at potential slip interface is divided by this coefficient. After a lot of trial calculations by adjusting the reduction coefficient until the critical failure state on the slip surface is reached, the coefficient is identified as the safety factor of stability against sliding. However, the rationality of this algorithm seems to be unreasonable for the following reasons.

Firstly, it is not quite reasonable for the strength parameters c and $\tan\varphi$ are divided by the same reduction coefficient. It is well known that c and $\tan\varphi$ are two independent parameters of shear strength in Mohr-Coulomb criterion. They have different physical meanings and play different roles. So, they shouldn't be divided by the same factor. If c and $\tan\varphi$ are respectively divided by different factors, the solution will become quite complicated and the number of combining solutions will become infinite. Moreover, when a potential sliding surface intersects with several zones with different materials, it is very unreasonable to still divide the strength parameters of different materials with the same reduction coefficient.

Secondly, the parameter φ can't be arbitrary reduced in Mohr-Coulomb criterion according to (Zheng et al. 2002). The Poisson's ratio μ and the friction angle φ have to follow the relationship

$$\sin\phi \geq 1 - 2\mu \tag{1}$$

This restricts the free reduction strength parameters.

Thirdly, when the strength parameters are reduced, the calculated stress state is not its real state, but a virtual state. The derivation from the virtual state is not physically sound. Additionally, the reduction region of strength parameters is still empirical in the finite element method. There is no rigorous theory to support the determination of reduction magnitude. When the stress state is close to its critical state due to the parameter reduction, the problem will become ill-conditioned. Consequently, it is very hard to obtain the reasonable solutions.

3 THE BASIC CONCEPT OF THE VECTOR SUM METHOD

Since force is vector, its sum should be vector sum. The resultant sliding force is the vector sum of the sliding forces of each segment Δl_i of the potential sliding surface. Similarly the resultant anti-sliding force is the sum of the anti-sliding forces of each segment Δl_i of the potential sliding surface.

When computing the safety factor, the resultant sliding force vector and the anti-sliding force vector must be projected to a certain direction. This projection direction should have clear physical meaning. In VSM the projection direction is defined as the potential direction of the potential sliding body. The safety factor is defined as the ratio of the projections in VSM.

Since the strength parameters do not need to be reduced in VSM, the stress state is the real stress state induced by the current external load, which can be easily obtained by certain FEM software or other numerical method. For the stability analysis by VSM is based on the real stress state and physical parameters, it has more advantages than those based on the virtual stress state. Moreover, the VSM analysis is based on the deformed body theory rather than the rigid body assumption. Therefore, VSM is more physical sound.

4 ANTI-SLIDING STABILITY ANALYSIS FOR 2D PROBLEM USING VSM

4.1 *Basic formula for 2D case*

For 2D problems, the calculating region, the potential sliding interface, and the sliding region are assumed to be prior known. The current stress state has been obtained by FEM or other methods. The search for critical slip surface is not included in this paper. Take Mohr-Coulomb criterion as the strength criterion. When the current stress states are known, the anti-sliding shear stress on the slip curve is calculated as

$$\tau_f = c + \sigma \tan\varphi \tag{2}$$

Take the potential sliding direction as the projection direction. The angle between potential sliding direction and the x coordinate axis is θ, shown in Figure 1.

According to definition of safety factor of VSM, the 2D safety factor K_{VS} is calculated as

$$K_{VS} = K(\theta) = \frac{\sum R(\theta)}{\sum T(\theta)} \tag{3}$$

where $\sum R(\theta)$ means the algebraic sum of projections on the potential sliding direction of anti-sliding force vector acting on the Δl_i; $\sum T(\theta)$ means the algebraic sum of projections on the potential sliding direction of sliding force acting on the Δl_i.

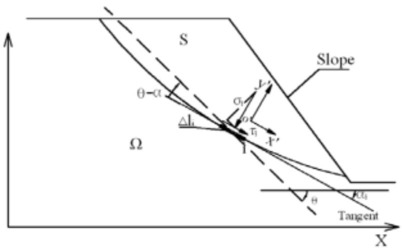

Figure 1. The coordinate systems and the stresses state acting on the Δl_i.

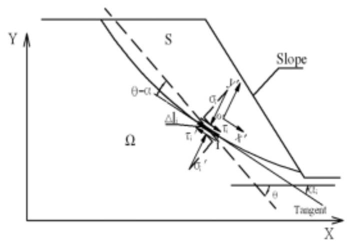

Figure 2. Diagram for solving safety factor by vector sum method.

4.2 *Determination of potential sliding direction*

Shown in Figure 1, take a segment Δl_i of the slip curve as the consideration object. Let Δl_i stand for the length of micro segment, σ_i and τ_i respectively for normal and shear stress of this micro segment, α_i for the inclination angle between x-axis and the tangent line at the point i on the slip surface of the global coordinates. The compressive is the positive for stress and the anti-clockwise for angle α_i is the positive. So, α_i in Figure 1 is negative.

According to Mohr-Coulomb criterion, the limit anti-sliding shear stress at the point i is calculated as

$$\tau_{fi} = c_i + \sigma_n tg\varphi_i \tag{4}$$

The opposite direction of resultant anti-sliding force is taken as the projection direction. Therefore, the projection angle θ is

$$\theta = \arctan\frac{\sum F_{yi}}{\sum F_{xi}} \tag{5}$$

where

$$F_{xi} = \tau_{fi}\Delta l_i \cos\alpha_i \tag{6}$$

$$F_{yi} = \tau_{fi}\Delta l_i \sin\alpha_i \tag{7}$$

4.3 *Analytical solutions of safety factor*

Figure 2 is a sketch for the solution of the safety factor by the Vector Sum Method. For a point i on the slip curve, the cohesion is c_i and internal frictional angle is φ_i. σ_i and τ_i are respectively the normal and the shear stress acting on the segment Δl_i, σ_i' and τ_i' are the normal stress and shear stress acting on the sliding mass by the bed rock. The relationships between them are

$$\sigma_i = \sigma_i',\ \tau_i = \tau_i' \tag{8}$$

For the sliding force, the projection of $\tau_i \Delta l_i$ is

$$T_\tau^i = \tau_i \Delta l_i \cos(\theta - \alpha_i) \tag{9}$$

and the projection of $\sigma_i \Delta l_i$ is

$$T_\sigma^i = \sigma_i \Delta l_i \sin(\theta - \alpha_i) \tag{10}$$

Therefore,

$$\sum T(\theta) = \sum T_\sigma^i + \sum T_\tau^i \tag{11}$$

The anti-sliding force is contributed by the cohesive and frictional forces at sliding interface. According to Mohr-Coulomb criterion, the anti-sliding force acted on segment Δl_i is calculated as

$$\tau_{fi} = c_i + \sigma_i \tan\phi_i \tag{12}$$

The projection of $\tau_{fi} \Delta l_i$ is

$$R_\tau^i = \tau_{fi} \Delta l_i \cos(\theta - \alpha_i) \tag{13}$$

The projection of $\sigma_i' \Delta l_i (= \sigma_i \Delta l_i)$ is

$$R_\sigma^i = \sigma_i \Delta l_i \sin(\theta - \alpha_i) \tag{14}$$

Therefore:

$$\sum R(\theta) = \sum R_\tau^i + \sum R_\sigma^i \tag{15}$$

Substituting Eqs. (11–15) into Eq. 3 yields

$$K_{VS} = K(\theta) = \frac{\sum R(\theta)}{\sum T(\theta)} = \frac{\sum R_\tau^i + \sum R_\sigma^i}{\sum T_\tau^i + \sum T_\sigma^i} \tag{16}$$

Eq. (16) can be further expanded as

$$K_{VS} = \frac{\sum_{i=1}^{n}\left[(c_i + \boldsymbol{\sigma}_i f_i)\cos(\theta-\alpha_i) + \boldsymbol{\sigma}_i \sin(\theta-\alpha_i)\right]\Delta l_i}{\sum_{i=1}^{n}\left[\boldsymbol{\tau}_i \cos(\theta-\alpha_i) + \boldsymbol{\sigma}_i \sin(\theta-\alpha_i)\right]\Delta l_i} \tag{17}$$

If σ_i is tensile stress, the frictional force $\sigma_i f_i$ on the slip length Δl_i should be zero.

4.4 *Validation of Vector Sum Method in 2D case*

To validate VSM, the standard examinations EX1 (a) and EX1(c) of the Association for Computer Aided Design, Limited (ACADS) (Chen 2003) are employed, shown in Figure 3 and Figure 4. Slope in EX1 (a) is a homogeneous, whose boundary conditions and dimensions are illustrated in Figure 3. The slope EX1(c) (Figure 4) is a nonhomogeneous, which is made up of 3 layers of soil. Its dimensions are the same with EX1 (a).

The results by different methods are listed in Table 1. From Table 1 it is seen that the calculated safety factors by VSM are in good agreement with the referee's answers. The relative errors between VSM and the recommended method (Donald) are only 1.06% for EX1(a) and 0.43% for EX1(c). For more examples and comparison analysis, refer to (Ge XR, 2009).

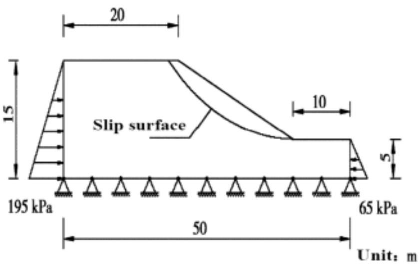

Figure 3. Calculating model EX1(a)of ACADS.

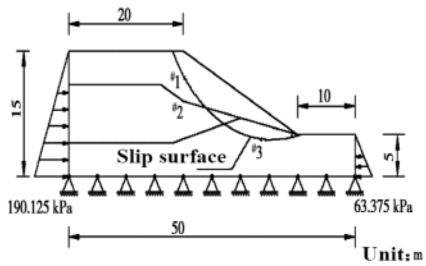

Figure 4. Calculating model EX1(c) of ACADS.

Table 1. Comparisons of safety factors for ACADS with different methods.

Analysis method	Program	EX1(a) Safety factor	EX1(c) Safety factor
Referee's answer with LEM	Donald (recommendation)	1.000	1.390
	SSA (Baker)	1.000	1.390
	STAB (Chen)	0.991	1.385
	GWEDGEM	1.000	1.390
	EMU	1.000	1.390
	Fredlund	0.990	1.406
VSM	ANSYS (elastic stress)	1.011	1.384

5 ANTI-SLIDING STABILITY ANALYSIS FOR 3D PROBLEM USING VSM

5.1 *Basic formula*

The safety factor in VSM is defined as the projection ratio of the resultant anti-sliding and sliding force vector on the potential sliding direction. The resultant sliding force is a vector sum of normal forces and shear forces acting on the ΔS_i of the potential slip surface. Similarly, the resultant anti-sliding force is a vector sum of anti-sliding normal force and anti-sliding shear force acting on the ΔS_i of the potential sliding surface by bed rock.

Shown as Figure 5, let $\boldsymbol{\sigma}_s$, $\boldsymbol{\sigma}_\tau$, $\boldsymbol{\sigma}_n$ stand for stress vector, shear stress and normal stress, respectively at the point A on the sliding interface. $\hat{n}$ is the unit normal vector of tangent plane at point A(positive pointing to outside of the sliding mass). $\hat{d}$ is the unit vector of the potential sliding direction. S is the slip surface. Then,

$$\boldsymbol{\sigma}_s = \boldsymbol{\sigma} \cdot \hat{\boldsymbol{n}} \tag{18}$$

$$\boldsymbol{\sigma}_n = (\boldsymbol{\sigma}_s \cdot \hat{\boldsymbol{n}})\ \hat{\boldsymbol{n}} \tag{19}$$

$$\boldsymbol{\sigma}_\tau = \boldsymbol{\sigma}_s - \boldsymbol{\sigma}_n \tag{20}$$

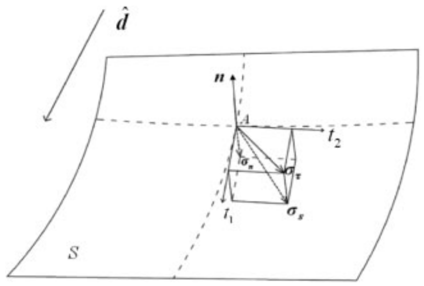

Figure 5. The stress state at point A on potential slip surface.

where σ is the stress tensor at the point A on the potential slip surface.

The normal stress acting on the sliding mass at the point A by bed rock is

$$\boldsymbol{\sigma}_n' = -\boldsymbol{\sigma}_n \tag{21}$$

To facilitate the derivation, assume that the tensile stress is positive and the compressive stress is negative. The safety factor is expressed:

$$K_{VS} = \frac{R}{T} \tag{22}$$

where R is the projection of resultant anti-sliding force on potential sliding direction $\boldsymbol{d}$; T is the projection of resultant sliding force on potential sliding direction $\hat{d}$ .They are

$$R = \int_s \boldsymbol{\sigma}_s' \cdot (-\hat{\boldsymbol{d}}) ds \tag{23}$$

$$T = \int_s (\boldsymbol{\sigma}_s \cdot \hat{\boldsymbol{d}}) ds \tag{24}$$

In Eq. 24, the limit anti-sliding stress vector $\boldsymbol{\sigma}_s'$ is

$$\boldsymbol{\sigma}_s' = \boldsymbol{\sigma}_\tau' + \boldsymbol{\sigma}_n' \tag{25}$$

If the Mohr-Coulomb criterion is accepted, the limit shear stress is calculated as

$$\boldsymbol{\sigma}_\tau' = (c - \sigma_n tg\varphi)\hat{\boldsymbol{d}}_r \tag{26}$$

where $\hat{d}_r$ is the unit direction vector of critical anti-sliding shear force on section dS of sliding surface; c cohesive force; φ the internal friction angle.

5.2 *Determination of unit direction vector $\boldsymbol{d_r}$ of limited anti-sliding shear force*

The determination of $\boldsymbol{d}_r$ is based on the maximum and minimum principles (Pan 1980; Chen 1998). According to this principle, $\boldsymbol{d}_r$ should take the opposite direction of the whole potential sliding direction at point A so that the maximum resistance against sliding can be rendered for a certain slip surface.

5.3 *Determination of whole potential sliding direction **d***

There always exists limit anti-sliding force at any point on the potential slip surface. According to the friction principle that the static friction direction is always opposite to trend of relative sliding, we define the potential sliding direction as

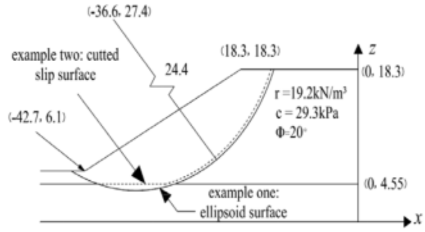

Figure 6. Model of the ellipsoidal example (Zhang 1988).

the opposite direction of the sum vector of limit anti-sliding forces, which is

$$\hat{\boldsymbol{d}} = \frac{-\int_s (\boldsymbol{c} - \sigma_n \boldsymbol{tg}(\varphi)) \hat{\boldsymbol{d}}_r ds}{\left\| \int_s (\boldsymbol{c} - \sigma_n \boldsymbol{tg}(\varphi)) \hat{\boldsymbol{d}}_r ds \right\|} \tag{27}$$

In Eq. (27), $\hat{d}_r$ is determined by the potential sliding direction and $\hat{d}$ can be obtained through $\hat{d}_r$. The implicit relationship between them can be established. The initial potential sliding direction is identified as

$$\hat{\boldsymbol{d}}_0 = \frac{\int_s \boldsymbol{\sigma}_\tau ds}{\left\| \int_s \boldsymbol{\sigma}_\tau ds \right\|} \tag{28}$$

where $\hat{d}_0$ is the initial potential sliding direction.

The following direction $\hat{d}_{r1}$ for each dS_i can be obtained by $\hat{d}_0$. Through Eq.27, the direction $\hat{d}_1$ can be calculated. It is a convergent sequence. The final potential sliding direction can be obtained through this iteration process until

$$\left\| \hat{\boldsymbol{d}}_{i-1} - \hat{\boldsymbol{d}}_i \right\| \le \varepsilon \tag{29}$$

where ε is toleration error for iteration, usually take $\varepsilon = 1.0\text{e-}4$.

5.4 *Validation of Vector Sum Method in 3D case*

To validate VSM in 3D case, three examples are presented. Two of them are from (Zhang 1988) in which a 3D ellipsoid example was presented. ***Example 1*** is a homogeneous slope, whose sliding surface is a part of simple ellipsoid surface; and in ***Example 2*** the upper part of sliding surface is the same as example 1, but the bottom of the slip surface is cut by a weak intercalation. Model of the ellipsoid examples is shown in Figure 6. The ***Example 3*** is an asymmetric wedge slope stability problem in rock mechanics.

Example 1, gravity is the only load acting on the slope with the two lateral sides normally restricted and the bottom fixed. The ideal elastic-plastic constitutive relationship of Mohr-Coulomb criterion and non-associated flow rule are adopted. Mesh scheme is shown in Figure 7. The total element number is 7094.

The safety factor calculated under elastic stress state is about 2.037 (see Table 2), which is slightly greater than that under the elastic-plastic stress state. Compared with other methods, the safety factors by VSM are reasonable and acceptable in practice.

Example 2: The ellipsoid is cut by a weak intercalation, and other conditions are the same as in Example 1 shown in Figure 8.

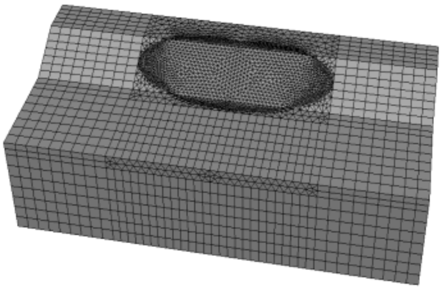

Figure 7. Meshes of the ellipsoidal model in example 1.

Table 2. Safety factors of different 3-D analysis methods in example 1

Zhang (1988)	Zheng (2007)	Chen (2001)	STAB-3D (Chen 2003)	VSM
2.122	2.140	2.262	2.188	2.037

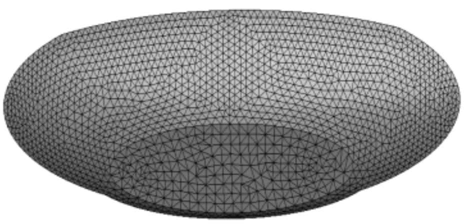

Figure 8. Cut slip surface in example 2.

Table 3. Safety factors of different 3D analysis methods in example 2 (EPS: Elasto-plastic state; ES: Elastic state).

Zhang (1988)	Hunger (1989)	Lam & Fredlund (1993)	Huang & Tsai (2000)	STAB-3D (Chen 2003)	VSM
1.553	1.620	1.603	1.658	1.640	1.545 (EPS) 1.585 (ES)

The safety factors by VSM and other methods are listed in Table 3. The safety factor by VSM is about 1.585 under the elastic stress state and 1.545 under the elastic-plastic stress state, which are consistent with the results by Zhang Xing.

Example 3: Stability of the wedge-shaped body is a typical 3-D slope stability problem in rock mechanics. It is an asymmetric geometry of wedge in this example. There is analytical solution about simple wedge by Limit Equilibrium Method, but it is based on the assumption that the shear stress direction on the slip surface is parallel to the crest line of two structure faces.

The parameters are: $E = 8.0e10$ Pa and $\upsilon = 0.3$, the density is 2600 Kg/m^3. Boundary conditions: slope surface and the top of wedge are free and other sides are restrained in their normal direction. The safety factor of the wedge by the Vector Sum Method is 1.654, and the analytical solution by limit equilibrium method is 1.64. It is consistent with each other very well.

5.5 *Application of VSM to engineering project*

The sketch of the selected dam foundation of a hydraulic power station in China is shown in Figure 9. The spatial distribution of

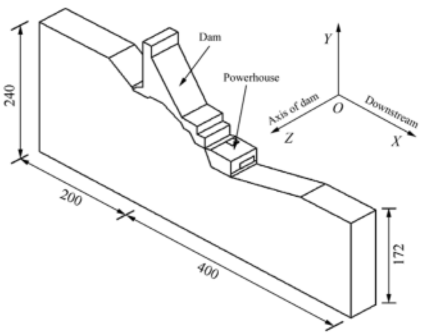

Figure 9. Geometric model of a dam foundation (unit: m).

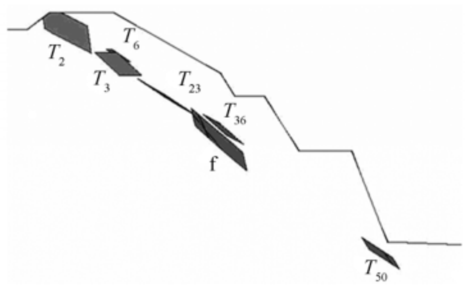

Figure 10. Spatial distribution of natural interfaces in the dam foundation.

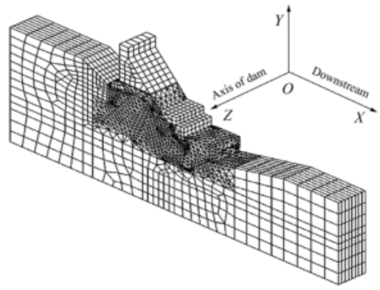

Figure 11. Meshing scheme of dam and foundation.

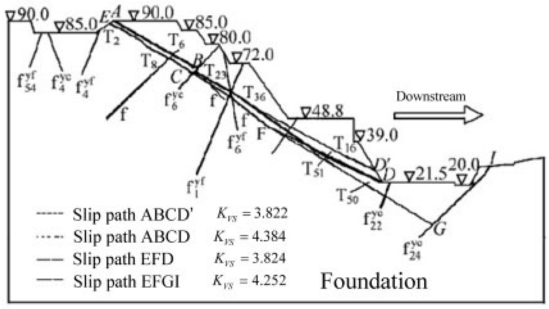

Figure 12. Main faults and potential slip paths in the profile of the dam foundation.

natural interfaces in the dam foundation is shown in Figure 10. The hexahedron element is adopted. The total element number is 47259 and the node number is 12590. The meshing scheme is shown in Figure 11. The main natural faults and the potential sliding interfaces of the representative section are shown in Figure 12. These sliding paths are ABCD′, ABCD, EFD and EFGI, as is shown in Figure 12. The corresponding results by 2D vector sum method are also listed in Figure 12.

In the 3D VSM analysis, the 3D sliding path is comprehensively determined by the sliding path of interface, the spatial

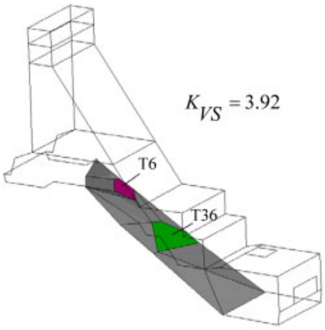

Figure 13. Sliding surface I in the dam foundation.

propagation of the embedded natural interfaces in dam foundation. In the present example, four 3D sliding interfaces are determined. Here only 3D sliding surface I and result are given in Figure 13.

6 CONCLUSIONS

The algorithm of safety factor in the limit equilibrium method is based on the strength reduction principle and the virtual stress state. The two features make the LEM and FEM-based method non-physical sound. To overcome the limitations of LEM and FEM-based method, the Vector sum method is proposed. For no parameters are reduced in VSM, the safety factor of VSM is derived from the real stress state rather than the virtual stress state. The safety factor is defined as the ratio of the projection of resultant anti-sliding and sliding force vector on the potential sliding direction, which has clear physical meaning. It can be calculated by an explicit expression in 2D case and by the iteration computation in 3D case. The simulation examples suggest that the safety factor is insensitive to the element size of FEM and the constitutive model (the elastic and elastic-plastic) in computation. The implementation of VSM is very simple and the iteration converges very fast for 3D case. The comparison analysis with other methods suggests that Vector Sum Method is reasonable and feasible. It provides a new method in stability analysis against sliding of slope and dam foundation engineering. This research is supported by National Key Technology R&D Program in the Eleventh-Five Year Plan of China (2008BAB29B03-3), for it we are much obliged.

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